Nanomachines: Methods to induce a directed motion at nanoscale

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The motion of bodies in a periodic potential with weak damping is discussed. A spontaneous directed motion of particles is shown to be possible in the presence of external periodic forces at a velocity unambiguously defined by the frequency of the periodic action and the spatial period of the potential. The principle of inducing the directed motion at a precisely controlled velocity can be used to develop (i) means of handling individual molecules or molecular clusters on crystalline surfaces; (ii) "nanomachines"—objects capable of spontaneous motion not only in the absence of an external force but also under the action of a force opposite to the direction of motion (and thereby capable of carrying other particles); (iii) drives providing precisely controlled velocity of motion; (iv) controllable tribological systems constructed by profiling friction surfaces in a specified manner and applying an ultrasonic excitation. The dependence of the average system velocity on the average applied force is shown to have plateaus of constant velocity at zero velocity and a set of equidistant discrete velocities in the presence of periodic external perturbations. The problem of developing fully controlled nanomachines can be formulated as the problem of controlling the width and position of the plateaus.

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I. INTRODUCTION

The tendency to miniaturization of mechanical systems and rapid development of nanotechnologies [1–5] pose a question about the theoretical limits of miniaturization. A fundamental problem in development of micromechanical systems at any level is the conversion of different types of energy to energy of directed motion of the system. Most ways of generating directed motion of molecular objects discussed in the literature [6–12] are based on the interaction between a driven object and an inhomogeneous, commonly periodically structured substrate. The latter can be either asymmetric or symmetric. Directed motion in asymmetric potentials has been discussed extensively in the context of molecular motors [8–10]. In this case, the directed motion is unidirectional; it is fixed by the interaction between the substrate and the object following the "ratchet-and-pawl" principle (see, e.g., [2,6,7]). In the case of symmetric potentials, the direction of motion originally is not fixed and is determined dynamically. An example of this kind of dynamically driven engine was given in recent publications [11,12]. In the present paper, we show that the nanoengine described in [11] is a special case of a wider class of systems of a different design, but with the same principles of control. In Sec. II, we consider motion of an object in a spatially periodic potential under the simultaneous action of a constant and an oscillating force, and discuss the dependence of the sliding velocity on the average driving force. The oscillating force gives rise to a specific feature of the force-velocity dependence: plateaus of constant velocity. In Sec. III, we show that these plateaus can be used for generating directed motion of objects in a spatially periodic potential under the action of oscillating forces; the controllability of the width and position of the plateaus is discussed. In Sec. IV, we show that it is possible to formulate simple topological rules determining the direction and velocity of motion of different systems under periodic actions. Section V is devoted to a discussion of possibilities to realize the nanomachines in practice.

II. MOTION IN A PERIODIC POTENTIAL UNDER THE ACTION OF PERIODIC FORCES

Let us consider one-dimensional motion of a body in a periodic potential with weak damping. The equation of motion for the body has the form

$$m\ddot{x} = F - \eta\dot{x} - N\sin(2\pi x/a), \tag{1}$$

where x is the body coordinate, m its mass, F the external force, η the damping factor, N the amplitude of the (spatially) periodic force, and a the wavelength of the periodic potential. The model was proposed by Tomlinson in 1929 as a model of dry friction [13]. In the general case, we assume that both the tangential force F and the amplitude N of the force acting from the periodic potential are periodic functions of time. In the following we are interested in determining the dependence of the average velocity $\langle\dot{x}\rangle$ of the mass ($\langle\cdots\rangle$ here denotes averaging over time) on the average force; after Tomlinson, we call this response function "the law of friction" of the system. In determining the friction law of the system two complementary approaches are possible; namely, we either specify the periodic force and pose the question of determining the average velocity of motion, or specify a periodically varying velocity and determine the average force required for maintaining the motion. The key features of the laws of friction are the same in both formulations. We start with the much simpler problem of determining the force at a given periodically varying velocity.

Let us assume that a periodic perturbation with amplitude v_1 is applied to the motion at a constant velocity v_0, so we have

$$v = v_0 + v_1\cos\omega t. \tag{2}$$

The coordinate of the body as a function of time is

$$x = x_0 + v_0 t + \frac{v_1}{\omega}\sin\omega t, \tag{3}$$

and the force acting on the body has the form

$$F=\eta(v_0+v_1\cos\omega t)+N\sin\frac{2\pi}{a}\left(x_0+v_0t+\frac{v_1}{\omega}\sin\omega t\right). \tag{4}$$

From here on we do not write the inertial force $m\ddot{x}$, since its average value is identically equal to zero.

To emphasize the basic features of the law of friction for this kind of motion, we start with the case where the dissipative force is absent and find the average value of the conservative part $\tilde{F}$ of the force (4):

$$\tilde{F}=N\sin\frac{2\pi}{a}\left(x_0+v_0t+\frac{v_1}{\omega}\sin\omega t\right). \tag{5}$$

The idea of the calculation can be followed most easily in the case where the amplitude of the velocity oscillations is much lower than the average sliding velocity, $v_1 \ll v_0$ (the case of arbitrary amplitude of oscillations is considered later). Given this assumption, the function (5) can be expanded in terms of the small parameter v_1/v_0:

$$\tilde{F}=N\left\{\sin\frac{2\pi}{a}(x_0+v_0t)+\frac{2\pi}{a}\cos\frac{2\pi}{a}(x_0+v_0t)\frac{v_1}{\omega}\sin\omega t\right\}. \tag{6}$$

Let us determine the time-averaged value of this force. Three cases are possible.

(1) If the average velocity is zero, $v_0=0$, the average force is

$$\langle F\rangle=N\sin\frac{2\pi}{a}x_0. \tag{7}$$

It can take any value in the interval $-|N|<\langle F\rangle<|N|$, and it evidently corresponds to the static frictional force.

(2) If the average velocity is nonzero, $v_0\neq 0$, and the condition $v_0\neq a\omega/2\pi$ is satisfied, the average force is identically equal to zero:

$$\langle F\rangle=0. \tag{8}$$

(3) Finally, if the velocity $v_0=a\omega/2\pi$, the average value of the first term in Eq. (6) is zero, and the second term results in a nonzero value of the average force:

$$\langle F\rangle=N\frac{\pi v_1}{a\omega}\sin\frac{2\pi}{a}x_0. \tag{9}$$

In this case, the force depends on the initial coordinate and can take an arbitrary value in the interval $-|N\pi v_1/a\omega|<\langle F\rangle<|N\pi v_1/a\omega|$ at a given average velocity of motion.

Thus, the law of friction appears as shown schematically in Fig. 1(a). The dissipative force omitted in the foregoing discussion will evidently lead only to uniform deformation of the plot in proportion to the velocity [Fig. 1(b)]. An essential feature of the law of friction in the presence of an external periodic action is the occurrence of plateaus of constant velocity not only at zero but also at finite velocities.

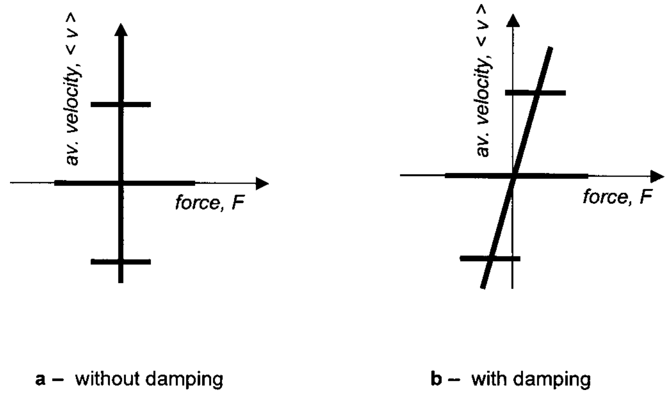

FIG. 1. Dependence of the average velocity of the particle on the average force in the case of a preassigned oscillating velocity (a) without and (b) with damping. The characteristic features of the velocity-force characteristic in the presence of a periodic external perturbation are *plateaus of constant velocity*.

The width of the plateaus depends on the amplitude of the periodic action. Details of this dependence will be discussed in the next section.

Obviously, for a plateau half-width larger than ηv_0, the plateau crosses the ordinate axis [as shown in Fig. 1(b)]. In this case, directed motion is possible for a zero average force or a force applied in the direction opposite to the direction of motion. In other words, an object can produce a tractive force, e.g., it can carry a cargo. It is this property that will be used below in designing nanomachines.

III. DYNAMIC NANOMACHINES

By "nanomachines" we mean tribological systems with a special form of the response characteristic "force–sliding velocity;" namely, it must exhibit plateaus of constant velocity whose width and positions may be controlled by external perturbations such that the system can be set in motion in an arbitrary direction. A system obeying the above law of friction [Fig. 1(b)] does not satisfy this definition yet, since its state of rest is stable. Although in the absence of an average force the body can move at a velocity corresponding to the first plateau, it should be given an initial impetus to initiate its motion. In the case of a controllable engine, it is desirable to eliminate the zero plateau completely. Thus, the state of rest would become unstable and the engine would spontaneously come into motion. Let us show first that it is possible to eliminate the zero plateau (and the static frictional force along with it) by varying the amplitude of the external periodic perturbation. To do so, we return to consideration of the action described by Eq. (2), but this time the perturbation is not assumed to be small. The average force (5) can be calculated analytically using the following expansions [14]:

$$\cos(\psi\sin\omega t)=J_0(\psi)+2\sum_{k=1}^{\infty}(-1)^kJ_{2k}(\psi)\cos(2k\omega t), \tag{10}$$

$$\sin(\psi\cos\omega t)=2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(\psi)\cos[(2k+1)\omega t], \tag{11}$$

where $J_n(\psi)$ is the nth-order Bessel function and

$$\psi=\frac{2\pi v_1}{a\omega}. \tag{12}$$

It can be shown that the average value of the force is identically equal to zero at any velocities except those satisfying the condition

$$v_0=\frac{\omega a}{2\pi}=V_0 n, \tag{13}$$

where V_0 is the velocity corresponding to the first plateau and n is integer. At velocities determined by Eq. (13), the average force is

$$\langle F\rangle=N(-1)^n J_n(v_1/V_0)\sin\frac{2\pi}{a}x_0. \tag{14}$$

For each discrete velocity (13), the force depends on the initial coordinate and can thus take an arbitrary value in a certain interval determined by the width of the corresponding plateau:

$$-|NJ_n(v_1/V_0)|\leqslant\langle F\rangle\leqslant|NJ_n(v_1/V_0)|. \tag{15}$$

As the oscillation amplitude increases, the zeroth-order plateau width decreases and eventually vanishes at a certain amplitude. This means that the static frictional force vanishes.

Our numerical simulations show that prescribing an oscillating force (instead of an oscillating velocity) leaves the key features of the force–sliding velocity characteristic unaffected. Let us consider a body that experiences a periodically oscillating force along with a constant tractive force F_0. The equation of motion has the form

$$m\ddot{x}=F_0+F_1\sin\omega t-\eta\dot{x}-N\sin(2\pi x/a). \tag{16}$$

A characteristic form of the average velocity as a function of the average applied force is shown in Fig. 2, where the result of a numerical solution to Eq. (16) is presented. The amplitude of the oscillating force component was chosen so that the zeroth plateau vanished. In this case, the static frictional force is absent. There is, however, equal probability that the body will move either in the positive or in the negative direction.

We will show that the *asymmetry of the force-velocity dependence and thus full controllability of the system* can be achieved by simultaneous oscillation of actions in the tangential and normal directions. Figure 3 shows the result of a numerical solution of the following equation describing the motion of a body that experiences oscillating, normal and tangential forces:

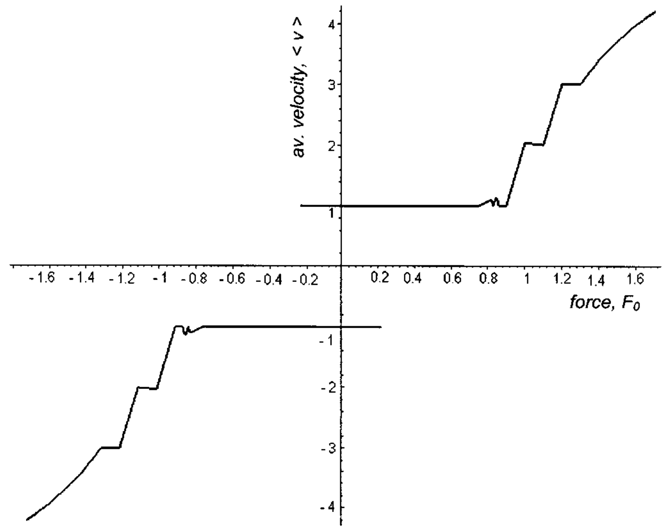

FIG. 2. Dependence of the average velocity of the particle on the average force under a periodically oscillating force derived by numerical solution to Eq. (16) for the following parameter values: $m=1$, $\omega=1$, $\eta=0.4$, $2\pi/a=1$, $F_1=2.3$. The plateaus partially overlap so that at slow increase and further decrease in the force there is hysteresis. The amplitude of oscillations was chosen so that the zeroth-order plateau was absent.

$$m\ddot{x}=F_0+F_1\sin\omega t-\eta\dot{x}-[N_0+N_1\sin(\omega t+\varphi_0)]\times\sin(2\pi x/a). \tag{17}$$

The oscillations of amplitude and phase shift were chosen so that the zeroth plateau vanished. In this case, the first and the negative first plateaus are asymmetrical so that in the absence of the average external force (or at low negative

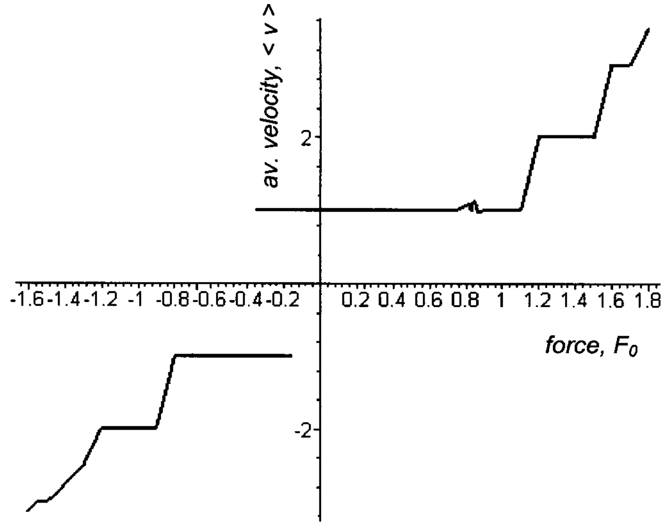

FIG. 3. Dependence of the average velocity of the particle on the average force for simultaneous oscillation of force and amplitude of the potential derived by numerical solution to Eq. (17) for the following parameter values: $m=1$, $F_1=2.3$, $\omega=1$, $N_0=1$, $N_1=0.6$, $\eta=0.6$, $\varphi_0=2.5$, $2\pi/a=1$. The parameters were chosen so that the zeroth-order plateau is absent.

forces) the body is capable of steady macroscopic motion only in the positive direction. The roles of the two plateaus can be changed by changing the phase shift φ_0.

IV. QUASISTATIC NANOMACHINES

In this section we will show that the frequency ω of perturbations for which the macroscopically ordered motion of the system is possible is not bounded from below. At low external perturbation frequency the motion can be considered to be quasistatic. In this case, analysis of the motion is reduced to examination of singular points and lines in the space of slowly varying system parameters (bifurcation sets). The feasibility of practical realization of an induced motion of nano-objects is discussed at the end of the paper.

A. Three-body system connected by rigid bonds

First let us examine the nanomachine proposed in [11]. In a simplified version, the machine represents a three-body system connected by rigid bonds l_1 and l_2 in length, in a spatially periodic harmonic potential. In this case, the total potential energy of the three bodies is

$$U=U_0(\cos[k(x-l_1)]+\cos(kx)+\cos[k(x+l_2)]), \tag{18}$$

where $k=2\pi/a$ and a is the spatial potential period. The potential energy (18) can be rewritten as

$$U=U_0\sqrt{(\sin kl_1-\sin kl_2)^2+(1+\cos kl_1+\cos kl_2)^2}\times\cos(kx-\varphi), \tag{19}$$

with

$$\tan\varphi=\frac{\sin kl_1-\sin kl_2}{1+\cos kl_1+\cos kl_2}. \tag{20}$$

The phase φ is a continuous and unambiguous function of the parameters l_1 and l_2 along any path on the (l_1,l_2) plane that does not pass through singular points where the potential amplitude (19) vanishes and the phase (20) is not defined. The position of these points is specified by the conditions $\sin kl_1-\sin kl_2=0$, $1+\cos kl_1+\cos kl_2=0$, or $kl_1=\pi\pm\pi/3+2\pi n$, $kl_2=\pi\pm\pi/3+2\pi m$, where m and n are integers. The positions of several singular points on the (l_1,l_2) plane are shown in Fig. 4. All the points are obtained by a periodic repetition of the two singular points $(kl_1,kl_2)=(2\pi/3,2\pi/3)$ and $(kl_1,kl_2)=(4\pi/3,4\pi/3)$ nearest to the origin of coordinates. Let us suppose that the lengths l_1 and l_2 can be controllably changed by some external action. When the first point in Fig. 4 is traced along a closed contour in the (l_1,l_2) plane (contour 1), the phase is decreased by 2π, and when the second point is traced in the same direction (contour 3), the phase is increased by 2π. Let us assign a topological index -1 to the first point and a topological index $+1$ to the second one. It is easy to verify that in the general case of the bond lengths varying along an arbitrary closed contour not passing through singular points, the phase is changed by $2\pi I$, where I is the sum of indices of all singular points enclosed by the contour. For example, contour 2 in Fig. 4 encloses no singular points; thus, in tracing around it, no final phase change occurs. Contour 4 encloses two singular points with indices -1; thus, in tracing along this contour the phase is changed by -4π. A change in the phase by a value of 2π means motion of a three-body system by one spatial period of the potential. This leads us to the suggestion that a periodic change in bond lengths l_1 and l_2 corresponding to the motion along any closed contour containing singular points with nonzero sums of indices will result in a macroscopic translational motion of the system. On the other hand, a change in bond length corresponding to a contour without any singular points or with singular points with a zero sum of indices will not cause any translational motion. If some contour on the (l_1,l_2) plane is traced periodically with a circular frequency ω, then the system will move with the macroscopic (average) velocity $v=(a\omega/2\pi)I$.

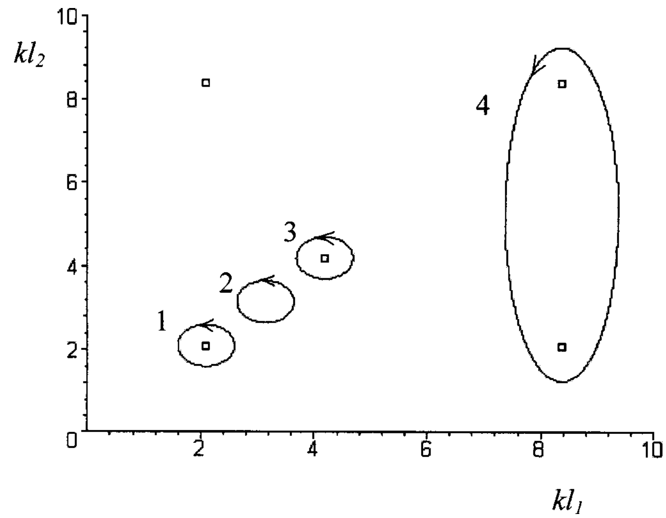

FIG. 4. Singular points of the potential (18) along with four different closed paths in the (l_1,l_2) plane. On changing the l_1 and l_2 lengths around the closed path 1 the phase is changed by -2π, in tracing around path 3 by $+2\pi$, in tracing the path 2 by zero, and in tracing the path 4 by -4π.

B. Two bodies connected by a bond

As a second example we consider a system of two bodies connected by a bond l in length in a periodic potential. The system under study is assumed to model an electrically polarized object on a crystalline surface. Application of an external electric field changes the system length and the forces of normal pressure acting on the two bodies (owing to the presence of the electric field component normal to the "substrate" surface). In this case, the normal forces acting on the first and second bodies are oppositely directed. This increases the periodic potential amplitude for one of the bodies and decreases it for the other. The system described can be modeled via the potential energy as

$$U=(N_0+N_1)\cos[k(x-l/2)]+(N_0+N_1)\cos[k(x+l/2)]$$
$$=2\sqrt{N_0^2\cos^2(kl/2)+N_1^2\sin^2(kl/2)}\cos(kx-\varphi) \tag{21}$$

with

$$\tan\varphi = (N_1/N_0)\tan(kl/2). \tag{22}$$

Here N_0 is the average periodic potential amplitude and N_1 is the change in the potential in response to the action of the electric field component normal to the surface.

A set of singular points for which the phase φ is not determined is given by the conditions $N_1=0$ and $\cos(kl/2)=0$. In the $(N_1/N_0,kl)$ plane, the set consists of the points $(0,\pi+2\pi n)$. A change in the phase by 2π, corresponding to translational motion of the system by one spatial period, occurs on tracing any of these points along a closed contour of an arbitrarily small radius.

C. Motion opposite to the force direction

The models considered above are of interest from the standpoint of development of methods for manipulating nano-objects, including those capable of carrying a load. Hence, an important problem is to find the conditions wherein the objects can move in the direction opposite to the average force acting on them. To examine the problem, we consider again a nanomachine consisting of three bodies in a harmonic potential in the presence of a constant external force $-F$ represented by the term Fx in the potential energy. In this case, the total potential energy has the form $U=U_0\{\cos[k(x-l_1)]+\cos(kx)+\cos[k(x+l_2)]\}+Fx$. Let us find a bifurcation set of this potential, defined as a set of parameter values for which a change in the number of equilibrium positions of the potential occurs (and thus, generally speaking, continuous and unambiguous dependence of the equilibrium position of the system on the control parameters is violated). Equilibrium positions are determined by the condition $\partial U/\partial x=0$. As a result of a bifurcation, a coalescence of stable and unstable equilibrium points occurs, whereupon both points disappear. At the bifurcation point itself, a vanishing stationary point is an inflection point, where $\partial^2 U/\partial x^2=0$. In the case under consideration, these two equations can be brought into the form

$$(1+\cos kl_1+\cos kl_2)^2+(\sin kl_1-\sin kl_2)^2=(F/U_0k)^2. \tag{23}$$

Figure 5 shows bifurcation sets determined by Eq. (23) for two values of the parameter $f=F/U_0k$. Translational motion is induced only in the case where the parameters l_1 and l_2 are changed along a contour that contains a closed bifurcation set originating from the former singular point. Evidently, this is possible only for $f<1$. Thus, the maximum theoretical tractive force of the engine under study is $F_{\max}=U_0k$. An analogous calculation for a two-body engine with potential energy $U=(N_0+N_1)\cos[k(x-l/2)]+(N_0-N_1)\cos[k(x+l/2)]$ gives for the maximum tractive force $F_{\max}=2N_0k$.

D. Nonpolarized particles

The foregoing systems are models of a polarized particle (dipole) in a variable electric field. Of greater practical importance, however, is the possibility of manipulating neutral and nonpolarized (but polarizable) particles. Let us formulate a model for a nanoparticle possessing electrostrictive properties and placed on a crystalline surface in a variable electric field that contains both tangential and normal components. Under the action of the field the particle changes its size. In the case of a particle consisting of a material whose spatial symmetry group contains an inversion center, the change in the particle size is a second-order effect in the field intensity. At the same time, the particle is polarized in proportion to the tangential electric field component, and a change of opposite sign in the pressure of the negatively and positively charged particle ends takes place. Such a system can be modeled by the potential energy (21) with

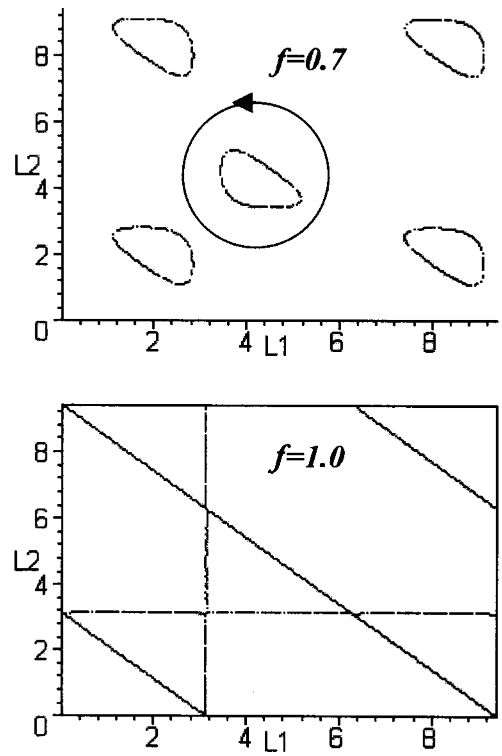

FIG. 5. Bifurcation sets of a three-particle engine in the presence of a constant external force. The path entirely enclosing the bifurcation set exists only for $f<1$.

$$l=l_0+\beta E^2 \quad \text{and} \quad N_1=\alpha E_\perp E_\parallel, \tag{24}$$

where $E_\perp$ and $E_\parallel$ are the normal and tangential components of the electric field, $E^2=E_\perp^2+E_\parallel^2$, β is the electrostrictive constant, and α is the particle polarizability. For an elliptically polarized variable electric field obeying the law $E_\perp=E_{\perp,0}\cos\omega t$, $E_\parallel=E_{\parallel,0}\sin\omega t$, we get

$$l=l_0+(\beta/2)(E_{\perp,0}^2+E_{\parallel,0}^2)+(\beta/2)(E_{\perp,0}^2-E_{\parallel,0}^2)\cos 2\omega t, \tag{25}$$

$$N_1=(\alpha/2)E_{\perp,0}E_{\parallel,0}\sin 2\omega t, \tag{26}$$

whence it follows that

$$\left(\frac{l-l_0-\beta(E_{\perp,0}^2+E_{\parallel,0}^2)/2}{\beta(E_{\perp,0}^2-E_{\parallel,0}^2)/2}\right)^2+\left(\frac{N_1}{\alpha E_{\perp,0}E_{\parallel,0}/2}\right)^2=1. \tag{27}$$

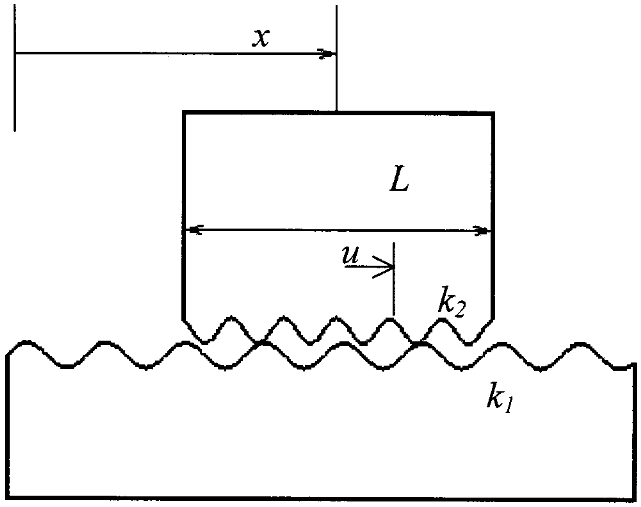

FIG. 6. A schematic representation of a nanoparticle with the crystal lattice constant $2\pi/k_2$ on a flat crystalline surface with period $2\pi/k_1$.

A set of values l and N_1 satisfying this equation forms an ellipse centered at the point $(l,N_1)=(l_0+(\beta/2)(E_{\perp,0}^2+E_{\parallel,0}^2),0)$. For $l_0+(\beta/2)(E_{\perp,0}^2+E_{\parallel,0}^2)=\pi+2\pi n$, the elliptical contour (27) encloses a singular point, and in tracing around it the system moves through a distance of one spatial period of the potential. Note that a change of one period in the electric field corresponds to two path tracings around the closed contour [as is evident from Eqs. (25) and (26)], so that the system moves through a distance of two spatial periods of the potential during one period of electric field oscillations.

How can the foregoing models be realized in practice? Let us examine a particle of length L and crystal lattice constant $2\pi/k_2$ located on a crystalline substrate with period $2\pi/k_1$ (Fig. 6). Let u be the current coordinate of points on the particle surface counted off from the center of mass of the particle and x be the coordinate of the center of mass of the particle in an external coordinate system. The interaction between the particle and the substrate is assumed to be so weak that deformation of the particle under the action of the interaction potential can be neglected. (Estimations show that this condition is satisfied for nanoparticles, provided the interaction is of the van der Waals type—for example, for particles of almost any "hard material" on a graphite surface.) To calculate the dependence of the particle potential energy on the coordinate of its center of mass, we write the charge density distribution over the particle surface as $q=q_0\cos k_2u$ and the potential distribution produced by the substrate as $\varphi=\varphi_0\cos k_1(x+u)$. The potential interaction energy between the particle and the substrate can be written as

$$\begin{aligned}U&=\int_{-L/2}^{L/2}q_0\varphi_0\cos k_2u\cos k_1(x+u)\,du\\&=q_0\varphi_0\cos k_1x(k_1+k_2)^{-1}\sin[(k_1+k_2)L/2]\\&\quad+(k_1-k_2)^{-1}\sin[(k_1-k_2)L/2].\end{aligned}\tag{28}$$

The singular points of the potential (28) are determined by the condition of vanishing of its amplitude, that is,

$$\begin{aligned}&(k_1+k_2)^{-1}\sin[(k_1+k_2)L/2]\\&\quad+(k_1-k_2)^{-1}\sin[(k_1-k_2)L/2]=0.\end{aligned}\tag{29}$$

Note that any changes of L due to electrostriction do not affect the product $k_2L/2=\pi L/a=\pi N$, which is just proportional to the number N of atoms in the contact region. The set of discrete values of particle lengths for which the potential amplitude vanishes is

$$k_1L=2\pi n,\tag{30}$$

where n is an integer. In tracing these singular points in the (L,N_1) plane around a closed path, the particle will travel a distance of one spatial period of the substrate. As shown earlier, this can be achieved by applying an electric field to the particle.

Let us discuss briefly the influence of thermal noise on the induction of directed motion of particles. In the case of a three-body machine, the main effect of changing the bond length is motion of the position of the potential minimum of the system. This can be most easily seen at a special choice of the phase shift of oscillations of both lengths: If we choose $l_1=(4/3)\pi/k+l_0\cos(\omega t)$ and $l_2=(4/3)\pi/k+l_0\cos(\omega t+\varphi)$ with $\varphi=(2/3)\pi$ and $l_0\ll 1/k$, then the potential energy (18) takes the form

$$\begin{aligned}&U_0kl_0[\sin(kx+\pi/3)\cos(\omega t+2\pi/3)-\sin(kx-\pi/3)\cos t]\\&\quad=U_0kl_0(\sqrt{3}/2)\cos(kx+\omega t+\pi/3).\end{aligned}$$

We then have to deal just with a potential of constant form propagating with constant velocity ω/k in the negative direction, that is with a "traveling potential ratchet" [6]. Neither inertial effects nor thermal fluctuations can change the character of the motion in this case: The average velocity always has the same sign as it would have without fluctuations, but is generally smaller in modulus. At a more general choice of phase shift, the three-body machine represents an object in a traveling and simultaneously pulsating potential. In the quasistatic case, due to the symmetry of the potential, the pulsation by itself cannot give rise to any directed motion. We thus suppose that—in the cases considered—the thermal noise does not disturb the directed motion.

V. CONCLUSION

In conclusion, let us estimate the order of magnitude of the minimum characteristic particle size with which the induction of directed motion of the particle on a crystalline substrate is possible. If the length of the particle is chosen equal to the critical value (30), an arbitrarily small periodic change of the length around this value combined with a polarization of the particle and action of a normal electric field will give rise to its directed motion. In this case the minimum particle size is that where the condition (30) is satisfied for the first time. However, even if the length of the particle is chosen by chance, the "distance" to the nearest singular point is in any case not larger than the lattice constant a of the substrate. Thus, oscillation of the particle length with an amplitude a combined with the action of the normal component of the electric field will lead to tracing around the nearest singular point and thus to a directed motion of the particle. The minimum size of the particle is then determined by the condition that the change in length of the particle under the action of an electric field with a realistic intensity should be about the lattice constant. As an example, consider a particle of $BaTiO_3$ (this material is chosen due to its large electrostrictive constant) at the surface of graphite. The electrostrictive deformation of $BaTiO_3$ under the action of an electric field with intensity $E=2\times 10^6$ V/m achieves $\varepsilon=4\times 10^{-4}$ [15]. The lattice constant of graphite is about $a=2.5$ Å. The condition for reaching the nearest singular point is $\Delta L=\varepsilon L\simeq a$. Thus directed motion is possible in any case for particles whose size is about (or larger than) 6×10^3 Å$=600$ nm. However, the motion should be possible even for smaller sizes, provided the length of the particle is chosen exactly equal (or very near) to the critical value (30).

Thus, the models considered in this paper show the possibility of directed motion of nanoparticles along a crystalline surface under the action of an oscillating elliptically polarized electric field. The direction of the particle motion is determined by the direction of polarization (and, thereby, by the direction of tracing singular points in the parameter plane). The magnitude of the velocity of motion can be controlled independently of its direction by the frequency of the applied periodic field.

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